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Properties of the scalar mesons $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$

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Abstract. In the three-state mixing framework, considering the possible glueball components of η and η', we investigate the hadronic decays of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ into two pseudoscalar mesons. The quarkonia–glueball content of the three states is determined from a fit to the new data presented by the WA102 Collaboration. We find that these data are insensitive to the possible glueball components of η and η'. Furthermore, we discuss some properties of the mass matrix describing the mixing of the isoscalar scalar mesons.

1 Introduction

Recently, based on the mass matrix motivated by [1], [2] has investigated the implications of the new data presented by the WA102 Collaboration [3] for the glueball–quarkonia content of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ (below denoted f_1, f_2 and f_3, respectively). We propose that some points can be improved on. First, in the reduced partial width $\Gamma(f_i \to \eta\eta')$ in [2], the sign of the contribution of the diagram d of Fig. 1 was flipped; it should be negative. The flipped sign actually arose from a typo in (A5) of [4] where the λ should all read $1/\lambda$[1]. Second, the mixing angle of η and η' was determined to have the very small value of $-5 \pm 4°$ [2] which is inconsistent with the value of $-15.5 \pm 1.5°$ as determined from a rather exhaustive and up-to-date analysis of data including strong decays of tensor and higher spin mesons, electromagnetic decays of vector and pseudoscalar mesons, and the decays of J/ψ [5]. Also, the possibility that glueball components exist in η and η' was not considered in [2]. Reference [6] already suggested that the η and η' wave functions need glueball components.

In this work, instead of the mass matrix by which one is confronted with confusing mass level order involving the masses of the bare states $(u\bar{u}+d\bar{d})/2^{1/2}$, $s\bar{s}$ and the glueball [1, 7–9], we shall adopt another mixing scheme which can be related to the mass matrix to describe the mixing of f_1, f_2 and f_3; then we can discuss some properties of the mass matrix based on our preferred results. In addition, we shall consider the possibility that the glueball components exist in η and η' when we investigate the hadronic decays of f_1, f_2 and f_3 into two pseudoscalar mesons, and check whether these new data are sensitive to the possible glueball components of η and η' or not.

2 Mixing scheme and decays

Based on the three Euler angles θ_1, θ_2 and θ_3, the mixing of f_1, f_2 and f_3 can be described as

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} a_8 & a_1 & a_g \\ b_8 & b_1 & b_g \\ c_8 & c_1 & c_g \end{pmatrix} \begin{pmatrix} |8\rangle \\ |1\rangle \\ |G\rangle \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}, \quad (1)$$

with

$$\begin{pmatrix} a_8 & a_1 & a_g \\ b_8 & b_1 & b_g \\ c_8 & c_1 & c_g \end{pmatrix} = \begin{pmatrix} c_1c_2c_3 - s_1s_3 & -c_1c_2s_3 - s_1c_3 & c_1s_2 \\ s_1c_2c_3 + c_1s_3 & -s_1c_2s_3 + c_1c_3 & s_1s_2 \\ -s_2c_3 & s_2s_3 & c_2 \end{pmatrix}, \quad (2)$$

$$\begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_2 & y_3 & z_3 \end{pmatrix} = \begin{pmatrix} a_8 & a_1 & a_g \\ b_8 & b_1 & b_g \\ c_8 & c_1 & c_g \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} & 0 \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

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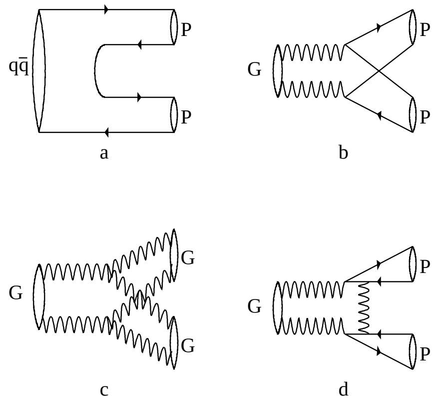

Fig. 1a–d. The coupling modes considered in this work. **a** The coupling of the quarkonia components of f_i to the final pseudoscalar meson pairs, **b** the coupling of the glueball components of f_i to the final pseudoscalar meson pairs via $qq\bar{q}\bar{q}$ intermediate states, **c** the coupling of the glueball components of f_i to the glueball components of the final isoscalar pseudoscalar mesons, and **d** the coupling of the glueball components of f_i to the quarkonia of the final isoscalar pseudoscalar meson pairs

where $|8\rangle = |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle/\sqrt{6}$, $|1\rangle = |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}$, $|N\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$, $|S\rangle = |s\bar{s}\rangle$, $|G\rangle = |gg\rangle$; c_1 $(c_2, c_3)\equiv \cos\theta_1$ $(\cos\theta_2, \cos\theta_3)$, s_1 $(s_2, s_3)\equiv \sin\theta_1$ $(\sin\theta_2, \sin\theta_3)$, and $-180^\circ \leq \theta_1 \leq 180^\circ$, $0^\circ \leq \theta_2 \leq 180^\circ$, $-180^\circ \leq \theta_3 \leq 180^\circ$. One advantage of this mixing model is the existence of only three unknown parameters with definite ranges of change.

Considering that glueball components possibly exist in the final isoscalar pseudoscalar mesons [6], for the hadronic decays of f_i (here and below, $i = 1, 2, 3$) into pseudoscalar meson pairs, we consider the following coupling modes as indicated in Fig. 1:

(a) the coupling of the $q\bar{q}$ components of f_i to the final pseudoscalar meson pairs;
(b) the coupling of the glueball components of f_i to the final pseudoscalar meson pairs via $qq\bar{q}\bar{q}$ intermediate states;
(c) the coupling of the glueball components of f_i to the glueball components of the final isoscalar pseudoscalar mesons; and
(d) the coupling of the glueball components of f_i to the $q\bar{q}$ components of the final isoscalar pseudoscalar meson pairs. Based on these coupling modes, the effective Hamiltonian describing the hadronic decays of f_i into two pseudoscalar mesons can be written as [10]

$$\begin{aligned} H_{\text{eff}} &= g_1 \mathrm{Tr}(f_F P_F P_F) + g_2 f_G \mathrm{Tr}(P_F P_F) + g_3 f_G P_G P_G \\ &\quad + g_4 f_G \mathrm{Tr}(P_F)\mathrm{Tr}(P_F), \end{aligned} \tag{4}$$

where g_1, g_2, g_3 and g_4 describe the effective coupling strengths of the coupling modes (a), (b), (c) and (d), respectively. f_G and P_G are SU(3) flavor singlets describing the glueball components of f_i and the final isoscalar pseudoscalar mesons, respectively. f_G and P_G can be given by

$$f_G = \sum_i z_i f_i, \quad P_G = \sum_j z_j j, \tag{5}$$

where z_j denotes the glueball content of j (here and below $j = \eta, \eta'$). f_F and P_F are 3×3 flavor matrixes describing the $q\bar{q}$ components of f_i and the final pseudoscalar mesons, respectively. f_F can be written as

$$f_F = \begin{pmatrix} \sum_i \frac{x_i}{\sqrt{2}} f_i & 0 & 0 \\ 0 & \sum_i \frac{x_i}{\sqrt{2}} f_i & 0 \\ 0 & 0 & \sum_i y_i f_i \end{pmatrix}, \tag{6}$$

P_F can be written as

$$P_F = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \sum_j \frac{x_j}{\sqrt{2}} j & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \sum_j \frac{x_j}{\sqrt{2}} j & K^0 \\ K^- & \overline{K^0} & \sum_j y_j j \end{pmatrix}, \tag{7}$$

where x_j and y_j denote the $(u\bar{u} + d\bar{d})/2^{1/2}$, $s\bar{s}$ contents of j, respectively, and they satisfy $x_j^2 + y_j^2 + z_j^2 = 1$.

Introducing $g_2/g_1 = r_1$, $g_3/g_1 = r_2$, $g_4/g_1 = r_3$, from (4)–(7) one can obtain

$$\Gamma(f_i \to \pi\pi) = 3 g_1^2 q_{i\pi\pi} [x_i + \sqrt{2} z_i r_1]^2, \tag{8}$$

$$\Gamma(f_i \to K\bar{K}) = g_1^2 q_{iK\bar{K}} [x_i + \sqrt{2} y_i + 2\sqrt{2} z_i r_1]^2, \tag{9}$$

$$\begin{aligned} \Gamma(f_i \to \eta\eta) &= g_1^2 q_{i\eta\eta} [x_\eta^2 x_i + \sqrt{2} y_\eta^2 y_i \\ &\quad + \sqrt{2}(x_\eta^2 + y_\eta^2) z_i r_1 + \sqrt{2} z_\eta^2 z_i r_2 \\ &\quad + (2\sqrt{2} x_\eta^2 + \sqrt{2} y_\eta^2 + 4 x_\eta y_\eta) z_i r_3]^2, \end{aligned} \tag{10}$$

$$\begin{aligned} \Gamma(f_i \to \eta\eta') &= g_1^2 q_{i\eta\eta'} [\sqrt{2} x_\eta x_{\eta'} x_i + 2 y_\eta y_{\eta'} y_i \\ &\quad + 2(x_\eta x_{\eta'} + y_\eta y_{\eta'}) z_i r_1 + 2 z_\eta z_{\eta'} z_i r_2 \\ &\quad + 2(2 x_\eta x_{\eta'} + \sqrt{2} x_\eta y_{\eta'} + \sqrt{2} x_{\eta'} y_\eta \\ &\quad + y_\eta y_{\eta'}) z_i r_3]^2, \end{aligned} \tag{11}$$

where $q_{iP_1P_2}$ is the decay momentum for the decay mode $f_i \to P_1 P_2$,

$$q_{iP_1P_2} = \frac{\sqrt{[M_i^2 - (M_{P_1} + M_{P_2})^2][M_i^2 - (M_{P_1} - M_{P_2})^2]}}{2M_i}, \tag{12}$$

M_i is the mass of f_i, M_{P_1} and M_{P_2} are the masses of the final pseudoscalar mesons P_1 and P_2, respectively, and we take $M_K = ((M_{K^\pm}^2 + M_{K^0}^2)/2)^{1/2}$.

For $\Gamma(f_i \to \eta\eta)$ and $\Gamma(f_i \to \eta\eta')$, the contribution of the coupling mode (d) given in our present work differs from that given in [2] since we do not adopt the assumption employed by [2] that the coupling of the glueball components of f_i to the $q\bar{q}$ components of the isoscalar pseudoscalar mesons occurs dominantly through their $s\bar{s}$ content in chiral symmetry. In addition, even under this assumption (i.e., x_j in the terms containing r_3 is set to be zero), for $\Gamma(f_i \to \eta\eta')$, the contribution of the mode (d) should be proportional to $+y_\eta y_{\eta'}$ but not $+2\alpha\beta \equiv -y_\eta y_{\eta'}$ as given by [2].

Table 1. The predicted and measured results of electromagnetic decays involving η, η'

	Exp. [12]	Fit 1 $z_j \neq 0$ $(j = \eta, \eta')$ $\chi^2 = 1.64$	Fit 2 $z_j = 0$ $(j = \eta, \eta')$ $\chi^2 = 9.19$
$\frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}$	58.46 ± 9.03	53.76	63.67
$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}$	540.78 ± 104.44	561.33	728.20
$\frac{\Gamma(\rho \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.051 ± 0.023	0.066	0.073
$\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.086 ± 0.016	0.086	0.111
$\frac{\Gamma(\phi \to \eta\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.078 ± 0.010	0.074	0.066
$\frac{\Gamma(\phi \to \eta'\gamma)}{\Gamma(\omega \to \pi^0\gamma)}$	0.0007 ± 0.0005	0.0003	0.0004
$\frac{\Gamma(J/\psi \to \rho\eta)}{\Gamma(J/\psi \to \omega\pi^0)}$	0.460 ± 0.120	0.482	0.533
$\frac{\Gamma(J/\psi \to \rho\eta')}{\Gamma(J/\psi \to \omega\pi^0)}$	0.250 ± 0.079	0.223	0.285

3 Fit results

Before performing the fit to determine the glueball–quarkonia content of f_i, we should first determine the parameters x_j, y_j and z_j. We will adopt the mixing scheme mentioned above to discuss the mixing of η, η' and $\eta(1410)$. Recently, the mixing of the three states based on a mass matrix has been discussed in [11]. Based on the (22)–(29) in Appendix A, the θ_1, θ_2 and θ_3 are determined as $\theta_1 = -98°$, $\theta_2 = 30°$ and $\theta_3 = -95°$, and x_j, y_j and z_j are determined as

$$x_\eta = -0.731, \quad y_\eta = 0.679, \quad z_\eta = -0.069,$$
$$x_{\eta'} = -0.566, \quad y_{\eta'} = -0.660, \quad z_{\eta'} = -0.495, \quad (13)$$

with $\chi^2 = 1.64$, which is consistent with the results given by [11]. If we set θ_2 and θ_3 to be zero, i.e., we do not consider the possible glueball components of j, the mixing angle of η and η' is determined to have a value of $-15°$, which is in good agreement with the value of $-15.5 \pm 1.5°$ given by [5], and x_j and y_j are determined to be

$$x_\eta = y_{\eta'} = [\cos(-15°) - \sqrt{2}\sin(-15°)]/\sqrt{3}, \quad (14)$$
$$x_{\eta'} = -y_\eta = [\sin(-15°) + \sqrt{2}\cos(-15°)]/\sqrt{3}, \quad (15)$$

with $\chi^2 = 9.19$. The χ^2 implies that the η and η' wave functions need additional glueball components. The predicted and measured results are shown in Table 1.

In order to investigate whether the new data given by [3] are sensitive to the possible glueball components of η and η' or not, we perform a fit to the data presented in Table 3 in two cases:

(a) $z_j \neq 0$ and
(b) $z_j = 0$.

In the fit procedure, we take $M_1 = 1.312$ GeV, $M_2 = 1.502$ GeV, $M_3 = 1.727$ GeV [3], and use the average value of 194 MeV for the decay momentum $q_{2\eta\eta'}$ [4], since f_2 lies very close to the threshold in the $\eta\eta'$ decay mode[2]. In fit (a) the parameters x_j, y_j and z_j are taken from (13) and in fit (b) x_j, y_j are taken from (14) and (15). The parameters θ_1, θ_2, θ_3, r_1, r_2 and r_3 in the two fits are determined as shown in Table 2 and the predicted and the measured results are shown in Table 3. Comparing fit (a) with fit (b), we find that the three Euler angles and the predicted results are not much altered, and that the χ^2 of the two fits are nearly equal, which shows that the new data on the hadronic decays of f_i into two pseudoscalar mesons are insensitive to the possible glueball components of η and η'.

Table 2. The parameters determined from the fit

	χ^2	r_1	r_2	r_3	θ_1	θ_2	θ_3
Fit (a)	2.05	1.0	3.4	0.33	$-146°$	$118°$	$-151°$
Fit (b)	2.15	1.0	0	0.7	$-148°$	$115°$	$-146°$

Table 3. The predicted and measured results of the hadronic decays of f_i

	Exp. [3]	Fit (a) $\chi^2 = 2.05$	Fit (b) $\chi^2 = 2.15$
$\frac{\Gamma(f_0(1370) \to \pi\pi)}{\Gamma(f_0(1370) \to K\bar{K})}$	2.17 ± 0.90	2.453	2.397
$\frac{\Gamma(f_0(1370) \to \eta\eta)}{\Gamma(f_0(1370) \to K\bar{K})}$	0.35 ± 0.30	0.248	0.314
$\frac{\Gamma(f_0(1500) \to \pi\pi)}{\Gamma(f_0(1500) \to \eta\eta)}$	5.56 ± 0.93	5.581	5.853
$\frac{\Gamma(f_0(1500) \to K\bar{K})}{\Gamma(f_0(1500) \to \pi\pi)}$	0.33 ± 0.07	0.335	0.308
$\frac{\Gamma(f_0(1500) \to \eta\eta')}{\Gamma(f_0(1500) \to \eta\eta)}$	0.53 ± 0.23	0.528	0.484
$\frac{\Gamma(f_0(1710) \to \pi\pi)}{\Gamma(f_0(1710) \to K\bar{K})}$	0.20 ± 0.03	0.191	0.200
$\frac{\Gamma(f_0(1710) \to \eta\eta)}{\Gamma(f_0(1710) \to K\bar{K})}$	0.48 ± 0.19	0.230	0.223
$\frac{\Gamma(f_0(1710) \to \eta\eta')}{\Gamma(f_0(1710) \to K\bar{K})}$	< 0.04(90%CL)	0.035	0.021

Based on the parameters with the lowest χ^2, the physical states $|f_1\rangle$, $|f_2\rangle$ and $|f_3\rangle$ can be given by

$$\begin{aligned} |f_1\rangle &= -0.599|N\rangle + 0.326|S\rangle - 0.732|G\rangle, \\ |f_2\rangle &= 0.795|N\rangle + 0.350|S\rangle - 0.495|G\rangle, \\ |f_3\rangle &= 0.095|N\rangle - 0.878|S\rangle - 0.469|G\rangle. \end{aligned} \quad (16)$$

From (16), one also can obtain

$$\Gamma(f_1 \to \gamma\gamma) : \Gamma(f_2 \to \gamma\gamma) : \Gamma(f_3 \to \gamma\gamma)$$

[2] In this paper, the values of the masses of other mesons are taken from [12]

$$= M_1^3(5x_1+\sqrt{2}y_1)^2 : M_2^3(5x_2+\sqrt{2}y_2)^2 : M_3^3(5x_3+\sqrt{2}y_3)^2 = 14.50 : 67.75 : 3.02. \quad (17)$$

This prediction can provide a test for the consistency of our results.

4 Discussions

Now we wish to discuss the properties of the mass matrix which can be used to describe the mixing of the scalar mesons based on our preferred results. In the $|N\rangle$, $|S\rangle$ and $|G\rangle$ basis, the general form of the mass matrix M describing the mixing of the quarkonia and a glueball can be written as [13]

$$M = \begin{pmatrix} M_N + 2A_1 & \sqrt{2}A_2 & \sqrt{2}B_1 \\ \sqrt{2}A_2 & M_S + A_3 & B_2 \\ \sqrt{2}B_1 & B_2 & M_G \end{pmatrix}, \quad (18)$$

where M_N, M_S and M_G represent the masses of the bare states $|N\rangle$, $|S\rangle$ and $|G\rangle$, respectively; A_1 (A_3) is the amplitude of $|N\rangle$ ($|S\rangle$) annihilation and reconstruction via intermediate gluons states; A_2 is the amplitude of the transition between $|N\rangle$ and $|S\rangle$; B_1 (B_2) is the amplitude of the transition between $|N\rangle$ ($|S\rangle$) and $|G\rangle$. If A_1, A_2 and A_3 are set to be zero, and B_1 is assumed to be equal to B_2, (16) would be reduced to the form employed in [1].

The physical states $|f_1\rangle$, $|f_2\rangle$ and $|f_3\rangle$ are assumed to be the eigenvectors of the mass matrix M with the eigenvalues of M_1, M_2 and M_3, then we can have

$$UMU^\dagger = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix},$$

$$\begin{pmatrix} |f_1\rangle \\ |f_2\rangle \\ |f_3\rangle \end{pmatrix} = U \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}. \quad (19)$$

Comparing (16) with (19), we have

$$U = \begin{pmatrix} -0.599 & 0.326 & -0.732 \\ 0.795 & 0.350 & -0.495 \\ 0.095 & -0.878 & -0.469 \end{pmatrix}. \quad (20)$$

Then the numerical form of the mass matrix is given by

$$M = U^\dagger \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} U = \begin{pmatrix} 1.436 & 0.018 & -0.093 \\ 0.018 & 1.656 & 0.138 \\ -0.093 & 0.138 & 1.450 \end{pmatrix}. \quad (21)$$

Equation (21) shows that A_2 is very small. If A_1 and A_3 also can be expected to be very small, the mass level order of the bare states $|N\rangle$, $|S\rangle$ and $|G\rangle$ would be $M_S > M_G > M_N$, which is consistent with the argument given by [2,8], while it disagrees with the prediction that the glueball state has a higher mass than the $q\bar{q}$ state [14]. Otherwise, the mass level order of M_N, M_S and M_G in the scalar sector would remain unclear. In addition, (21) implies that the mass of the pure scalar glueball is about 1.5 GeV, which is consistent with the lattice QCD prediction [15].

A salient property of (21) is that $B_1 < 0$ and $B_2 > 0$. This shows that the amplitude of the transition between $|N\rangle$ and $|G\rangle$ is negative while the amplitude of the transition between $|S\rangle$ and $|G\rangle$ is positive, which disagrees with the assumption that $B_1 = B_2$ in Weingarten's model [1]. This difference maybe results from the fact that Weingarten's model [1] does not consider SU(3) flavor breaking corrections, i.e., the possibility that the conversion of gluon into $q\bar{q}$ may have a significant flavor dependence [16], as well as some possible contributions induced by non-perturbative effects. In fact, the assumption suggested by chiral symmetry that the direct coupling of a gluon to the η or η' occurs dominantly through their $s\bar{s}$ component [2,17] also implies that the conversion of the gluon into $q\bar{q}$ may be flavor dependent. However, all these additional corrections exist in (18), and therefore exist in x_i, y_i and z_i since the mixing scheme mentioned in Sect. 2 in effect is equivalent to (18). Strictly speaking, the effective Hamiltonian (4) should contain an explicit SU(3) breaking term; however, since x_i, y_i and z_i already contain the SU(3) breaking corrections, we assume that the contribution from the explicit SU(3) breaking term in the effective Hamiltonian (4) can be ignored. Therefore, although the effective Hamiltonian (4) is constructed in the SU(3) limit, our analysis still allows for the SU(3) breaking corrections. Of course, our result that B_1 and B_2 are out of phase is different from the general expectation in lattice QCD that B_1 and B_2 are in phase; however, in the present situation, lattice QCD predictions maybe still have remaining options open due to the quenched approximation etc. In the absence of detailed understanding of the dynamics mechanism of the conversion of gluon into different $q\bar{q}$ and the lattice QCD presenting incontrovertible conclusions on this matter, there might not be convincing reasons to expect that the relation between B_1 and B_2 should behave as $B_1 = B_2$.

In addition, based on Weingarten's model [1], one can get different predictions on the quarkonia–glueball structure of f_i due to the different assumptions about the mass level order of M_G, M_S and M_N [1,2,7–9]. That our present results are different from the previous results [1,2,7–9] mainly results from that the fact that our analysis does not make any assumption on the mass level order of M_G, M_S and M_N, and also from the assumption that $B_1 = B_2$ is not incorporated in our present analysis.

We note that the values of r_1 and r_2 are inconsistent with r_1 and r_2 being less than unit, the prediction given by the perturbative theory. We find that if we restrict

r_1, r_2 and r_3 to the viewpoint of the perturbative theory, i.e., $r_1 < 1$, $r_2 < 1$ and $r_3 < 1$, the χ^2 increases from 2.05 to 3.80, but the results given above are not much altered. However, in the scalar sector there are no convincing reasons to expect that the perturbative theory should be valid. The values of r_1 and r_2 imply that the non-perturbative effects in the scalar sector could be rather large.

5 Summary and conclusions

Using the three Euler angles, we introduce a mixing scheme to describe the mixing of the isoscalar scalar mesons. In this mixing framework, considering the four coupling modes as shown in Fig. 1, we construct the effective Hamiltonian to investigate the two-body hadronic decays of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. The glueball–quarkonia content of the three states is determined from the fit to the new data about the hadronic decays of the three states presented by the WA102 collaboration. Our conclusions are as follows.

(1) A large mixing effect exist in the three states.
(2) The new data on the hadronic decays of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ are insensitive to the possible glueball components of η and η'.
(3) The non-perturbative effects in the scalar sector are rather large.
(4) Our preferred results do not support the assumption employed by Weingarten's mass matrix describing the mixing of the isoscalar scalar states [1] that $B_1 = B_2$.

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Appendix A: Formulae for the electromagnetic decays widths rates involving η and η'

The relevant formulae are

$$\frac{\Gamma(\eta\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)} = \frac{1}{9}\left(\frac{M_\eta}{M_{\pi^0}}\right)^3 (5x_\eta+\sqrt{2}y_\eta)^2, \tag{22}$$

$$\frac{\Gamma(\eta'\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)} = \frac{1}{9}\left(\frac{M_{\eta'}}{M_{\pi^0}}\right)^3 (5x_{\eta'}+\sqrt{2}y_{\eta'})^2, \tag{23}$$

$$\frac{\Gamma(\rho\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)} = \left[\frac{(M_\rho^2-M_\eta^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_\rho}\right]^3 x_\eta^2, \tag{24}$$

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\omega\to\pi^0\gamma)} = 3\left[\frac{(M_{\eta'}^2-M_\rho^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_{\eta'}}\right]^3 x_{\eta'}^2, \tag{25}$$

$$\frac{\Gamma(\phi\to\eta\gamma)}{\Gamma(\omega\to\pi^0\gamma)} = \frac{4}{9}\frac{m_u^2}{m_s^2}\left[\frac{(M_\phi^2-M_\eta^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_\phi}\right]^3 y_\eta^2, \tag{26}$$

$$\frac{\Gamma(\phi\to\eta'\gamma)}{\Gamma(\omega\to\pi^0\gamma)} = \frac{4}{9}\frac{m_u^2}{m_s^2}\left[\frac{(M_\phi^2-M_{\eta'}^2)M_\omega}{(M_\omega^2-M_{\pi^0}^2)M_\phi}\right]^3 y_{\eta'}^2, \tag{27}$$

$$\frac{\Gamma(J/\psi\to\rho\eta)}{\Gamma(J/\psi\to\omega\pi^0)} = \left[\frac{\sqrt{[M_{J/\psi}^2-(M_\rho+M_\eta)^2][M_{J/\psi}^2-(M_\rho-M_\eta)^2]}}{\sqrt{[M_{J/\psi}^2-(M_\omega+M_{\pi^0})^2][M_{J/\psi}^2-(M_\omega-M_{\pi^0})^2]}}\right]^3 \times x_\eta^2, \tag{28}$$

$$\frac{\Gamma(J/\psi\to\rho\eta')}{\Gamma(J/\psi\to\omega\pi^0)} = \left[\frac{\sqrt{[M_{J/\psi}^2-(M_\rho+M_{\eta'})^2][M_{J/\psi}^2-(M_\rho-M_{\eta'})^2]}}{\sqrt{[M_{J/\psi}^2-(M_\omega+M_{\pi^0})^2][M_{J/\psi}^2-(M_\omega-M_{\pi^0})^2]}}\right]^3 \times x_{\eta'}^2, \tag{29}$$

where M_ρ, M_ω, M_ϕ and $M_{J/\psi}$ are the masses of ρ, ω, ϕ and J/ψ, respectively; m_u and m_s are the masses of the constituent quark u and d, respectively. Here we take $m_u/m_s = 0.642$ as used in [18].

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